

Review Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Sketching a Graph In Exercises 1 and 2, sketch the graph of the function and state its domain.

- $f(x) = \ln x - 3$
- $f(x) = \ln(x + 3)$

Expanding a Logarithmic Expression In Exercises 3 and 4, use the properties of logarithms to expand the logarithmic expression.

- $\ln \sqrt[5]{\frac{4x^2 - 1}{4x^2 + 1}}$
- $\ln[(x^2 + 1)(x - 1)]$

Condensing a Logarithmic Expression In Exercises 5 and 6, write the expression as the logarithm of a single quantity.

- $\ln 3 + \frac{1}{3} \ln(4 - x^2) - \ln x$
- $3[\ln x - 2 \ln(x^2 + 1)] + 2 \ln 5$

Finding a Derivative In Exercises 7–12, find the derivative of the function.

- $g(x) = \ln \sqrt{2x}$
- $f(x) = \ln(3x^2 + 2x)$
- $f(x) = x \sqrt{\ln x}$
- $f(x) = [\ln(2x)]^3$
- $y = \ln \sqrt{\frac{x^2 + 4}{x^2 - 4}}$
- $y = \ln\left(\frac{4x}{x - 6}\right)$

Finding an Equation of a Tangent Line In Exercises 13 and 14, find an equation of the tangent line to the graph of the function at the given point.

- $y = \ln(2 + x) + \frac{2}{2 + x}, \quad (-1, 2)$
- $y = 2x^2 + \ln x^2, \quad (1, 2)$

Finding an Indefinite Integral In Exercises 15–18, find the indefinite integral.

- $\int \frac{1}{7x - 2} dx$
- $\int \frac{x^2}{x^3 + 1} dx$
- $\int \frac{\sin x}{1 + \cos x} dx$
- $\int \frac{\ln \sqrt{x}}{x} dx$

Evaluating a Definite Integral In Exercises 19–22, evaluate the definite integral.

- $\int_1^4 \frac{2x + 1}{2x} dx$
- $\int_1^e \frac{\ln x}{x} dx$
- $\int_0^{\pi/3} \sec \theta d\theta$
- $\int_0^{\pi} \tan \frac{\theta}{3} d\theta$

Finding an Inverse Function In Exercises 23–28, (a) find the inverse function of f , (b) graph f and f^{-1} on the same set of coordinate axes, (c) verify that $f^{-1}(f(x)) = x$ and $f(f^{-1}(x)) = x$, and (d) state the domains and ranges of f and f^{-1} .

- $f(x) = \frac{1}{2}x - 3$
- $f(x) = 5x - 7$
- $f(x) = \sqrt{x + 1}$
- $f(x) = x^3 + 2$
- $f(x) = \sqrt[3]{x + 1}$
- $f(x) = x^2 - 5, \quad x \geq 0$

Evaluating the Derivative of an Inverse Function In Exercises 29–32, verify that f has an inverse. Then use the function f and the given real number a to find $(f^{-1})'(a)$. (Hint: Use Theorem 5.9.)

- $f(x) = x^3 + 2, \quad a = -1$
- $f(x) = x\sqrt{x - 3}, \quad a = 4$
- $f(x) = \tan x, \quad -\frac{\pi}{4} \leq x \leq \frac{\pi}{4}, \quad a = \frac{\sqrt{3}}{3}$
- $f(x) = \cos x, \quad 0 \leq x \leq \pi, \quad a = 0$

Solving an Exponential or Logarithmic Equation In Exercises 33–36, solve for x accurate to three decimal places.

- $e^{3x} = 30$
- $-4 + 3e^{-2x} = 6$
- $\ln \sqrt{x + 1} = 2$
- $\ln x + \ln(x - 3) = 0$

Finding a Derivative In Exercises 37–42, find the derivative of the function.

- $g(t) = t^2 e^t$
- $g(x) = \ln \frac{e^x}{1 + e^x}$
- $y = \sqrt{e^{2x} + e^{-2x}}$
- $h(z) = e^{-z^2/2}$
- $g(x) = \frac{x^2}{e^x}$
- $y = 3e^{-3/t}$

Finding an Equation of a Tangent Line In Exercises 43 and 44, find an equation of the tangent line to the graph of the function at the given point.

- $f(x) = e^{6x}, \quad (0, 1)$
- $f(x) = e^{x-4}, \quad (4, 1)$

Implicit Differentiation In Exercises 45 and 46, use implicit differentiation to find dy/dx .

- $y \ln x + y^2 = 0$
- $\cos x^2 = x e^y$

Finding an Indefinite Integral In Exercises 47–50, find the indefinite integral.

- $\int x e^{1-x^2} dx$
- $\int x^2 e^{x^3+1} dx$
- $\int \frac{e^{4x} - e^{2x} + 1}{e^x} dx$
- $\int \frac{e^{2x} - e^{-2x}}{e^{2x} + e^{-2x}} dx$

Evaluating a Definite Integral In Exercises 51–54, evaluate the definite integral.

51. $\int_0^1 xe^{-3x^2} dx$

52. $\int_{1/2}^2 \frac{e^{1/x}}{x^2} dx$

53. $\int_1^3 \frac{e^x}{e^x - 1} dx$

54. $\int_0^2 \frac{e^{2x}}{e^{2x} + 1} dx$

55. Area Find the area of the region bounded by the graphs of $y = 2e^{-x}$, $y = 0$, $x = 0$, and $x = 2$.



56. Depreciation The value V of an item t years after it is purchased is $V = 9000e^{-0.6t}$ for $0 \leq t \leq 5$.

- Use a graphing utility to graph the function.
- Find the rates of change of V with respect to t when $t = 1$ and $t = 4$.
- Use a graphing utility to graph the tangent lines to the function when $t = 1$ and $t = 4$.

Sketching a Graph In Exercises 57 and 58, sketch the graph of the function by hand.

57. $y = 3^{x/2}$

58. $y = \left(\frac{1}{4}\right)^x$

Finding a Derivative In Exercises 59–64, find the derivative of the function.

59. $f(x) = 3^{x-1}$

60. $f(x) = 5^{3x}$

61. $y = x^{2x+1}$

62. $f(x) = x(4^{-3x})$

63. $g(x) = \log_3 \sqrt{1-x}$

64. $h(x) = \log_5 \frac{x}{x-1}$

Finding an Indefinite Integral In Exercises 65 and 66, find the indefinite integral.

65. $\int (x+1)5^{(x+1)^2} dx$

66. $\int \frac{2^{-1/t}}{t^2} dt$

67. Climb Rate The time t (in minutes) for a small plane to climb to an altitude of h feet is

$$t = 50 \log_{10} \frac{18,000}{18,000 - h}$$

where 18,000 feet is the plane's absolute ceiling.

- Determine the domain of the function appropriate for the context of the problem.
- Use a graphing utility to graph the time function and identify any asymptotes.
- Find the time when the altitude is increasing at the greatest rate.

68. Compound Interest

- How large a deposit, at 5% interest compounded continuously, must be made to obtain a balance of \$10,000 in 15 years?
- A deposit earns interest at a rate of r percent compounded continuously and doubles in value in 10 years. Find r .

Evaluating an Expression In Exercises 69 and 70, evaluate each expression without using a calculator. (*Hint:* Make a sketch of a right triangle.)

69. (a) $\sin(\arcsin \frac{1}{2})$

70. (a) $\tan(\operatorname{arccot} 2)$

(b) $\cos(\arcsin \frac{1}{2})$

(b) $\cos(\operatorname{arcsec} \sqrt{5})$

Finding a Derivative In Exercises 71–76, find the derivative of the function.

71. $y = \tan(\arcsin x)$

72. $y = \arctan(2x^2 - 3)$

73. $y = x \operatorname{arcsec} x$

74. $y = \frac{1}{2} \arctan e^{2x}$

75. $y = x(\arcsin x)^2 - 2x + 2\sqrt{1-x^2} \arcsin x$

76. $y = \sqrt{x^2 - 4} - 2 \operatorname{arcsec} \frac{x}{2}, \quad 2 < x < 4$

Finding an Indefinite Integral In Exercises 77–82, find the indefinite integral.

77. $\int \frac{1}{e^{2x} + e^{-2x}} dx$

78. $\int \frac{1}{3 + 25x^2} dx$

79. $\int \frac{x}{\sqrt{1-x^4}} dx$

80. $\int \frac{1}{x\sqrt{9x^2 - 49}} dx$

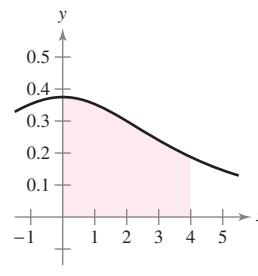
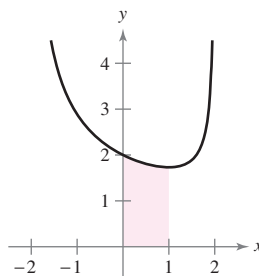
81. $\int \frac{\arctan(x/2)}{4+x^2} dx$

82. $\int \frac{\arcsin 2x}{\sqrt{1-4x^2}} dx$

Area In Exercises 83 and 84, find the area of the region.

83. $y = \frac{4-x}{\sqrt{4-x^2}}$

84. $y = \frac{6}{16+x^2}$



Finding a Derivative In Exercises 85–90, find the derivative of the function.

85. $y = \operatorname{sech}(4x - 1)$

86. $y = 2x - \cosh \sqrt{x}$

87. $y = \coth(8x^2)$

88. $y = \ln(\cosh x)$

89. $y = \sinh^{-1}(4x)$

90. $y = x \tanh^{-1} 2x$

Finding an Indefinite Integral In Exercises 91–96, find the indefinite integral.

91. $\int x^2 \operatorname{sech}^2 x^3 dx$

92. $\int \sinh 6x dx$

93. $\int \frac{\operatorname{sech}^2 x}{\tanh x} dx$

94. $\int \operatorname{csch}^4(3x) \coth(3x) dx$

95. $\int \frac{1}{9-4x^2} dx$

96. $\int \frac{x}{\sqrt{x^4-1}} dx$